STRUCTURAL HEALTH MONITORING IN COMPOSITE MATERIALS USING FREQUENCY RESPONSE METHODS

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ABSTRACT

Cost-effective and reliable damage detection is critical for the utilization of composite materials in structural applications. Non-destructive evaluation techniques (e.g. ultrasound, radiography, infra-red imaging) are available for use during standard repair and maintenance cycles, however by comparison to the techniques used for metals these are relatively expensive and time consuming. This paper presents part of an experimental and analytical survey of candidate methods for the detection of damage in composite materials. The experimental results are presented for the application of modal analysis techniques applied to rectangular laminated graphite/epoxy specimens containing representative damage modes, including delamination, transverse ply cracks and through-holes. Changes in natural frequencies and modes were then found using a scanning laser vibrometer, and 2-D finite element models were created for comparison with the experimental results. The models accurately predicted the response of the specimens at low frequencies, but the local excitation and coalescence of higher frequency modes makes mode-dependant damage detection difficult and most likely impractical for structural applications. The frequency response method was found to be reliable for detecting even small amounts of damage in a simple composite structure, however the potentially important information about damage type, size, location and orientation were lost using this method since several combinations of these variables can yield identical response signatures.

Keywords: Composites, Modal Analysis, Damage Detection, Structural Health Monitoring, Non-Destructive Testing

1. INTRODUCTION

Structural Health Monitoring (SHM) has been defined in the literature as the "acquisition, validation and analysis of technical data to facilitate life-cycle management decisions." [1] More generally, SHM denotes a reliable system with the ability to detect and interpret adverse "changes" in a structure due to damage or normal operation. The greatest challenge in designing a SHM system is to identify what "changes" to look for. Several techniques have been researched for detecting these "changes" in composite materials, many of them focusing on modal response [2-21]. These methods are among the earliest and most common, principally because they are simple to implement on any size structure. Structures can be excited by ambient energy, an external shaker or embedded actuators, and embedded strain gauges, piezos or accelerometers can be used to monitor the structural dynamic responses. Changes in normal vibrational modes can be correlated to loss of stiffness in a structure, and usually analytical models or experimentally determined response-history tables are used to predict the corresponding location of damage [22]. The difficulty, however, comes in the interpretation of the data collected by this type of system. There are also detection limitations imposed by the resolution and range of the individual sensors chosen, and the density with which they are distributed over the structure. There have been many different approaches described in the literature that use modal evaluation techniques to locate damage in everything from small specimens to full components.

One of the most thorough reports can be found in a recently published paper by Zou *et al.* [22], which presents a review of vibration-based techniques that rely on models for identification of delamination in composite structures. The authors suggest that model-dependent methods are capable of providing both global and local damage information, as well as being cost-effective and easily operated. All of the methods they assessed use piezo sensor and actuators along with finite element analysis results to locate and estimate damage events by comparing changes in dynamic responses. The paper compares the

merits of four different dynamic response parameters: modal analysis, frequency domain, time domain and impedance domain. The authors recommended modal analysis methods on account of their global nature, low cost, and flexibility to select measurement points, however they indicated that they lack the ability to detect very small damage and require large data storage capacity for comparisons. They claimed that frequency domain methods alone were incapable of detecting the location of damage, however when combined with time domain methods they can detect damage events both globally and locally. Lastly, the impedance domain techniques were described as suitable for detecting most delamination reliably, unless the layers above the defect are very thin compared to the remaining laminate.

Zhang *et al.* [23] investigated the use of transmittance functions for health monitoring, a technique which does not require the use of analytical models. A system of piezo patches are placed on a structure, where some are used as sensors and others as actuators, and responses at certain sensor locations due to broadband actuation are recorded for the healthy structure to be later compared to a potentially damaged structure. Changes in curvature were used in this case to detect, locate and assess damage to the structure. One significant finding was that the optimal range to actuate their structure was between 10-20 kHz, however they found that only frequencies less than 5 kHz (200-1800 Hz in most cases) were practical to collect experimental data. Lastly, Valdes and Soutis [24] detected delamination in composite laminates using piezoceramic patches and piezoelectric film sensors, again without the use of models. Frequency sweeps between 8-14 kHz were used to induce vibrations on the structure, and clear reductions in modal frequencies were found as the delamination area in the test specimen was increased.

This present paper specifically investigates the feasibility of modal evaluation techniques in detecting damage for health monitoring of composite structures. Characteristics examined include these methods' ability to detect various types of damage, their precision in determining the damage location, and their sensitivity to sensor density. The impact of conformability for system implementation is also assessed. A similar procedure is followed by several of the papers presented above to evaluate these methods. Composite laminated coupons were manufactured, and several representative forms of damage were introduced to these specimens. Experiments were performed to detect the various forms of damage with different sensor systems. Finite element models were then created to validate these results and to perform trade-studies. Finally a discussion is presented remarking on the potential role of frequency response methods in a SHM system.

2. EXPERIMENTAL SETUP

Four graphite/epoxy panels were manufactured according to standard in-house procedure [25] using AS4/3501-6. A $[90/\pm45/0]_s$ quasi-isotropic laminate was selected for these experiments, and the specimens were cut to 250 x 50 x 1 mm using a diamond grit cutting wheel. Next, various types of damage were introduced to the specimens. In the first group, 6.4 mm diameter holes were drilled into the center of each specimen using a silicon-carbide core drill to minimize damage during the drilling process. The next group was impacted with a mallet. The third group was compressively loaded in a 4-point bending fixture until audible damage was heard, and the fourth was cyclically loaded in the same fixture for 2000 cycles at 80% of this load with an R ratio of -1. The next two groups of specimens were delamination specimens. Two methods were used to introduce the delamination: one used a thin utility blade to cut a 50 x 20 mm slot in one side, and the other with a Teflon strip cured into the center of the laminate. In both cases the delamination was at the center mid-plane of the laminate. The final group consisted of the control specimens. After the damage was introduced into each specimen an x-ray radiograph was taken using a die-penetrant to help document the type, degree and location of the damage as shown in **Figure 1**.

In order to deduce the natural frequencies and mode shapes of the specimens, a scanning laser vibrometer system from $Polytec^{TM}$ was used. The clamped boundary condition simulating a cantilever beam was found to be the most sensitive factor in the experiments, so the specimens were clamped to a pre-specified load of 9 N-m in a vice using a torque wrench. The specimens were excited using two square 13 mm PZT wafers which were temporary adhered with thin double-sided tape to the base of the specimen and actuated out of phase by an 8 V sine chirp signal (fast repeated sine sweep [26]), which was sent to the piezos through a function generator to drive them between 0 Hz and 20 kHz. A separate set of tests was also performed using an external shaker to excite the specimens with this same chirp signal. The laser was set to scan through a fine mesh of points along each specimen's surface recording the velocity response at that grid position. This data along with complimentary data from a stationary control laser were used to produce frequency/response plots and mode shapes.

3. FINITE ELEMENT MODEL

A 2-D finite element analysis was performed in I-DEASTM to determine the frequency response of graphite/epoxy composite specimens. Eight-node quadrilateral shell elements were used (500 in total) to model the 250 x 50 x 1 mm specimen. A convergence study was performed to determine that 6 mm square elements were optimal to solve for the normal modes of the system, with a change in resonant frequencies of less than 0.1% by decreasing the element size by 1 mm². To simulate a clamped boundary condition, the nodes on the bottom 25 mm of the specimen were constrained in all of their degrees of freedom. A Classical Laminated Plate Theory (CLPT) [27] code was written in MATLABTM to calculate the composite elastic matrices for a $[90/\pm45/0]_s$ quasi-isotropic laminate of AS4/3501-6 (E₁ = 142 GPa, E₂ = 9.8 GPa, G₁₂= 5.4 GPa, v₁₂= 0.3), which were then entered into a material property card in I-DEASTM. The "Simultaneous Vector Iteration" method was used to calculate the natural frequencies of the system up to 20 KHz, and their corresponding mode shapes.

Several types of damage were also simulated in various models, as represented in **Figure 2**. One simple variation of the control model had a hole modeled into it. Other models had altered extension and bending stiffness matrices either in specific regions or across the entire model, which simulated reduction in axial stiffness due to distributed damage caused by either static or fatigue loading as suggested by the literature. For transverse ply cracks in a quasi-isotropic laminate caused by a static load, the results of two studies showed that the stiffness is reduced asymptotically to 90-95% of its original value as the specimen reaches saturation [28, 29]. These same studies found that the laminate modulus is affected more by fatigue-induced cracks for the same crack density, achieving about 80% of its original value. The last damage model simulated a delaminated area in the specimen. This was accomplished by calculating separate sets of elastic matrices and assigning the appropriate new properties to each half laminate in the delaminated area.

4. RESULTS

4.1 Experimental Results

There were three sets of outputs for each test on the vibrometer. The velocity magnitude response to the frequency range inputted into the piezos was recorded by the vibrometer system. The vibrometer software was used to compute the normal mode maximum peaks and corresponding deformation shapes. Again the dynamic responses were found between 0-20 kHz, a sample of which for the vibrometer results of a control specimen is shown in **Figure 3**. A table comparing the first six natural frequencies and mode-shapes of a control specimen and several other damaged specimens (as described in the experimental setup) can be found in **Table 1**. A few selected mode shapes from the vibrometer display are presented in **Figure 4** to be later contrasted to the predicted shapes. Lastly, **Figure 5** displays the velocity magnitude response to a frequency range below 500 Hz for all of the tested specimens. From this plot, the conclusions regarding the true effect of various damage on the frequency response of a system can be extracted.

4.2 Model Response

As with the experimental results, there are three sets of results that are presented in this paper that were generated by I-DEASTM for each model. The first is a list of natural frequencies converged to a specified number of significant figures for the frequency range requested. The second is a series of plots of the mode shapes that correspond to these natural frequencies. The final result is a transfer function plot for the velocity magnitude response to the frequency spectrum. As suggested by the literature, all of these results were found in the range of 0-20 kHz, and the transfer function plot of this range is shown in **Figure 6**. From this plot it was apparent that not much data could be visually extracted from such a broad frequency range, so the rest of the data presented here are for the modes below 500 Hz with an explanation of this decision in the discussion section. A table comparing the first six natural frequencies and mode-shapes of each specimen can be found in **Table 2**. This particular simulated damage model had a 25 x 50 mm delamination located in the mid-plane of the specimen along the free edge, and was modeled as described previously. A few graphical samples of their mode-shapes can be seen in **Figure 7**. The most relevant set of results found from the analytical part of this research was the transfer function comparison plot as exemplified in **Figure 8**. Several other similar plots were generated in 1-DEASTM for the other damage mode models, all yielding similar trends. The significance of these plots will be delineated in the discussion section.

5. DISCUSSION

5.1 Effect of Damage on Frequency Response

For both the numerical (FE) and experimental results it is evident that all the forms of damage investigated in this study caused detectable changes in the natural frequencies of a simple coupon. These changes are present in each of the lower normal frequencies discovered, and become more pronounced at higher frequencies to a degree that corresponding modes between the control and damaged specimens become indiscernible. This frequency reduction can be explained by classical structural dynamics [30]. Natural frequencies are determined by the boundary conditions of a system through the variable λ^2 , which is determined by the characteristic equation of the structure, and is multiplied by the ratio (EI/m)^{0.5}, where E = modulus, $I = 2^{d}$ moment of area and m = mass. When damage is introduced to a specimen by one of a variety of mechanisms, the resulting local loss of stiffness directly affects this ratio, thereby affecting the natural frequencies of the structure. The delaminated specimens have a region that effectively behaves as two separate laminates with reduced stiffness, and the set with one edge delaminated has an even more prominent change in the torsion modes due to its asymmetry. The fatigue-damaged specimens are affected by matrix-cracking and fiber-matrix debonding, and the 4-point bending specimens contain broken fibers, which also reduce the modulus. Changes in the specimens with the drilled hole can be explained by the reduced stiffness and inertia.

As shown in the literature, a strong correlation can often be found between relative frequency reduction and the area damaged by a particular mechanism, however it is difficult to draw any conclusions about the criticality of the damage since there is no information regarding the form of the damage or its orientation. This limitation is illustrated by a delamination with an area of 50 x 20 mm that has a significantly different effect on a structure depending on whether the longer delamination direction is oriented parallel or perpendicular to the sensor. Delamination that is more severe along the length of a specimen tends to cause a larger reduction on the bending modes, while delamination along the width appears to impact the torsional modes more adversely. It is also important to note that the 5-10% reduction in natural frequency caused by a 6.4 mm diameter through-hole yields almost identical transfer function results to that of a 50 x 50 mm center delamination, however there is a noteworthy difference in the significance to the structure. The only type of damage that was slightly distinguishable at low frequency ranges was fatigue damage, which produced many high-energy local modes that were not present in any of the other specimens. Based on these results, it is likely that an observer can discern whether a structure has been damaged by observing its frequency response, however it would be difficult to differentiate reliably between damage types, locations and orientations without capturing several accurate bending and torsional modes and building a large database of damage simulations model and experimental data.

5.2 Comparison of Tests to Model

In comparing the test results to those solved in IDEASTM, a good correlation was found without tuning or adjusting the model. The models consistently yielded the correct progression of mode shapes, and natural frequencies around 1-8% above those found in experiments, which could be explained by a variety of factors. A small amount of error could be attributed to fiber misalignment and resin flow or bleed-out during curing that created slight differences in the modulus, density and thickness of the laminate, affecting the natural frequencies by a factor of $(Et^2/\rho)^{0.5}$, which is a manipulated version of the previous constant where t = thickness and ρ = density. The experimentally obtained averages of these variables were identical to those entered into the model, however significant variances are associated with them, which would account for as much as a 4% deviation from the predicted solution. Another possible difference could have been introduced by the actuating piezos, which add non-negligible mass to the specimens and may also have shifted the measurement equipment slightly out of phase due to the elasticity in the thin adhesive layer between them and the composite laminate. The largest variable sensitivity in the system was found to be in the simulated clamped boundary condition. It was experimentally found that by slightly loosening the clamp, the lowest natural frequencies in the control specimen could drop as much as 10%, which overshadowed most forms of damage that was detected in these specimens. This result was confirmed by a finite element model, which replaced the completely clamped boundary condition with a pin on the 25 mm line and a clamp at the base of the specimen. This model yielded a 9% reduction in the first several resonant frequencies. Consequently, much care was taken to produce a consistent clamping pressure with the torque wrench for each of the experiments performed, however the inability to simulate an ideal clamp, as evident by an observed slope at the clamp, most likely contributed to the majority of the experimental error.

The strong dependence on accurate boundary conditions to retrieve accurate frequency responses for even a simple geometry model is the reason why most work in the literature has avoided model-dependant SHM solutions. Instead, they have tended towards time history change comparisons while using this technique. Without the use of models however, the frequency response method is limited to low frequency ranges where the response peaks are still distinct. A consequence of this limitation is that while the principal global modes can be detected, the local modes of the structure, which hold the most detailed information about the damage present, will not be detected. Even so, as will be discussed in the following section, the frequency response method can still play an important role in a SHM system, and preliminary models are useful in predicting the response of a structure to help design a successful sensor layout.

5.3 Role of Frequency Response Methods in SHM

There are many advantages to using a frequency response method in a SHM system; they can be implemented cheaply, they can be light and conformal, and they can provide good insight as to the global condition of the system. The limitations are that they provide little information about the local damage area unless large quantities of sensors are used along with accurate numerical models; and then it can be argued that damage large enough to be detected globally may already be critical in many structures. Clearly this method does not suffice as the sole sensor set in a SHM system, however that does not exclude them from having any role in the system. The most attractive implementation of the frequency response method is one performed passively for low frequencies using ambient vehicle vibrations, caused by the engines or aerodynamic loads for example. Comparing global transfer functions for prescribed frequency ranges at selected positions could provide a good foundation for a first and last line of defense in a SHM system. A passive method such as this could continuously monitor components of a structure without requiring much processing power in order to direct more accurate and energy-intensive active sensor systems where to query for a more detailed survey of potential damage. Alternatively, widespread fatigue may be too small or gradual to be detected by fine-tuned active methods, and may be better detected by an ambient frequency response method by setting a global limit on allowable natural frequency decay of the structure over time. To accomplish this role in a SHM system, first a model would have to be built which would be used to select an appropriate range of frequencies to clearly detect resonant frequencies and to test various placements for sensors. Modal reduction can be accomplished using a variety of sensors such as strain gauges, piezoceramic wafers or accelerometers, which must be placed strategically throughout the structure. Then a damaged model should be used to confirm that realistic damage would be detected from the transfer function for the selected frequency range. Lastly these results should be experimentally verified on a representative structure, perhaps by increasing local stiffness externally instead of damaging the structure. In non-critical and smaller components this method may also prove sufficient to detect most forms of damage.

6. CONCLUDING REMARKS

The potential role of the frequency response of a composite structure in a Structural Health Monitoring system has been investigated in this paper. A finite element model was built to numerically explore the effects of various types of damage on the normal modes of test coupons, and analogous experiments were performed using a scanning laser vibrometer and impedance meter to verify these results. Good correlation was found between the model and the experimental results for low frequencies, however coalescing modes at higher frequencies made comparison impractical. In both the numerical model and experimental results there was strong correspondence between the extent of damaged (or local stiffness loss) and reduction in natural frequency. The limitations and sensitivities of the frequency response method are discussed as well. This method appears to be appropriate for detecting global changes in stiffness, and hence damage, for relatively large structures at a low power and weight cost. Further research interests lie in the implementation of this method with discrete sensors, and a comparison of the sensitivity of results using various sensors. Future work will focus on a similar study for different detection methods, such as ones that use embeddable sensors to detect lamb-waves or eddy-currents. The final goal of this research is to provide useful guidelines in sensor selection and system architecture for designing a reliable SHM system for composite structures.

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- Matrix-crack specimen with fatigue induced damage c.
- d. Delamination specimen cut with a thin utility knife at the mid-plane



Delamination specimen with two laminate groups in damaged area d.



| (All Hz) | Shape | Control | Hole | Impact | Delamination | Fatigue | Bend | | | | |
|--|-------------------------|---------|------|--------|--------------|---------|------|--|--|--|--|
| Mode 1 | 1 st Bending | 12.5 | 12.5 | 12.5 | 12.5 | 12.5 | 12.5 | | | | |
| Mode 2 | 2 nd Bending | 78.1 | 78.1 | 76.5 | 78.1 | 75.0 | 76.3 | | | | |
| Mode 3 | 1 st Torsion | 157 | 148 | 147 | 137 | 146 | 137 | | | | |
| Mode 4 | 3 rd Bending | 218 | 217 | 216 | 215 | 209 | 214 | | | | |
| Mode 5 | 4 th Bending | 423 | 423 | 423 | 428 | 413 | 423 | | | | |
| Mode 6 | 2 nd Torsion | 461 | 453 | 453 | 451 | 428 | 432 | | | | |
| Table 1: Natural frequencies and mode shapes as determined from scanning laser vibrometer data | | | | | | | | | | | |







| (All Hz) | Shape | Control | Hole | Impact | Delamination | Fatigue | Bend | | | |
|--|-------------------------|---------|------|--------|--------------|---------|------|--|--|--|
| Mode 1 | 1 st Bending | 12.5 | 12.4 | 12.5 | 12.1 | 12.1 | 12.3 | | | |
| Mode 2 | 2 nd Bending | 77.8 | 77.2 | 77.5 | 75.5 | 73.7 | 76.3 | | | |
| Mode 3 | 1 st Torsion | 157 | 155 | 156 | 149 | 150 | 154 | | | |
| Mode 4 | 3 rd Bending | 218 | 217 | 217 | 211 | 213 | 216 | | | |
| Mode 5 | 4 th Bending | 428 | 425 | 426 | 412 | 413 | 422 | | | |
| Mode 6 | 2 nd Torsion | 476 | 473 | 474 | 465 | 466 | 472 | | | |
| Table 2: Natural frequencies and mode shapes as determined from FEM in I-DEAS. | | | | | | | | | | |



