Analytical Axisymmetric Coupled Piezo-Elastodynamic Models for Guided-Wave Structural Health Monitoring

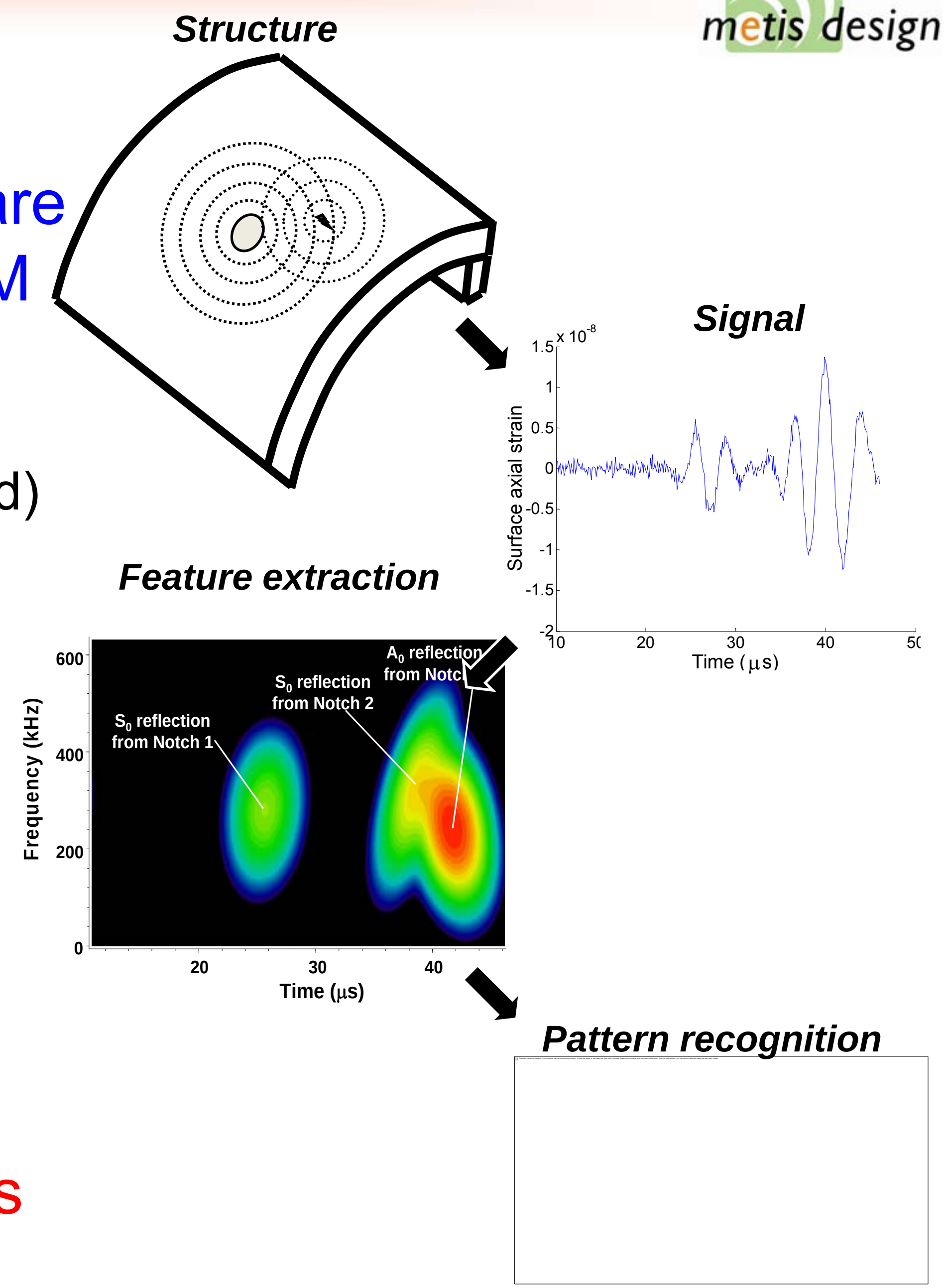
Christopher T. Dunn, Ajay Raghavan & Seth S. Kessler

Metis Design Corporation

10 Canal Park • Cambridge, MA 02141 • 617.661.5616 • http://www.metisdesign.com

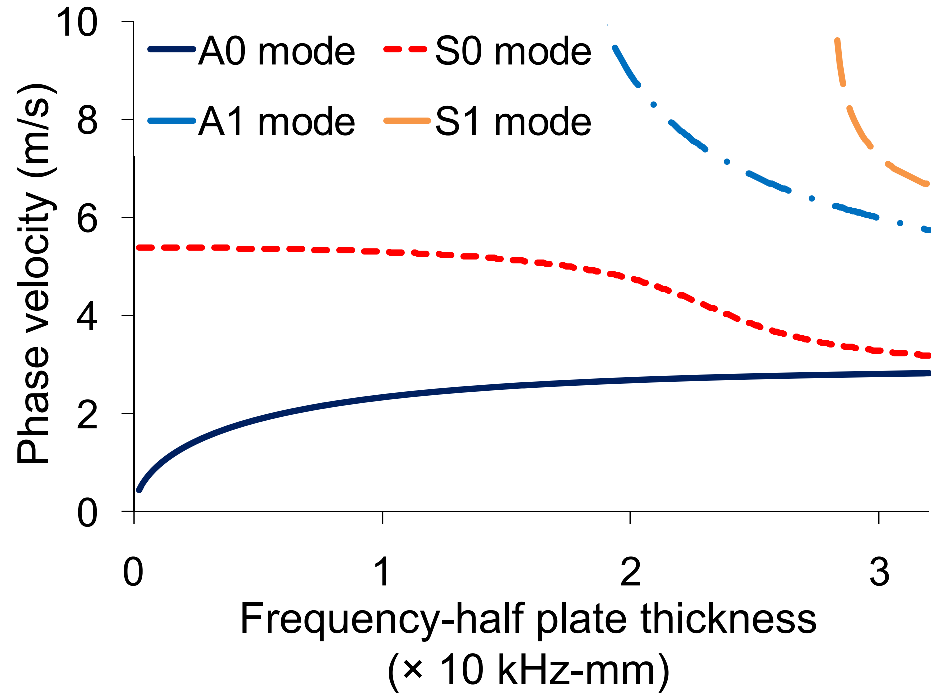
Introduction

- Guided-waves (Lamb waves) are lead method for wide-area SHM
- Advantages:
  - active method (checks on demand)
  - extensive propagation range
  - short inspection time
  - accurately triangulate & diagnose damage using sparse network
- Piezoelectric transducers most commonly used for GW
- Strong need for physics-based piezoelectric / structural models



Characteristics of Guided Waves

- Guided waves
  - stress waves forced to follow a path defined by the material boundaries of a structure
  - multi-modal, each mode has a unique dispersion curve
- Wave speeds & excitability
  - frequency-dependent
  - directional if anisotropic
- Coupling dynamics adds further complexity
- Reliable model needed for effective SHM system design



Phase speed dispersion curve for aluminum plates

Previous Analytical Efforts

- Majority focused on uncoupled actuator/structural models
  - effective for low actuator-structure thickness ratios (≤ 10%)
  - require calibration for actuator traction magnitude estimate
  - cannot capture piezoelectric resonance effects
  - e.g., Lin and Yuan (2001), Veidt et al. (2004), Rose and Wang (2004), Giurgiutiu (2003), Raghavan and Cesnik (2005/2007)

- Few efforts for coupled piezoelectric actuator/substrate models
  - Zagrai and Giurgiutiu (2001): Coupled piezoelectric-structural (Mindlin theory) model for electromechanical impedance
  - Moulin et al. (2000)/Bottai et al. (2007): 2-D plane strain coupled FE/analytical guided wave models for transducers bonded on plates
  - Glushkov et al. (2006): Higher fidelity 2-D plane strain guided wave analysis of transducers on plates using integro-differential equations

Objectives

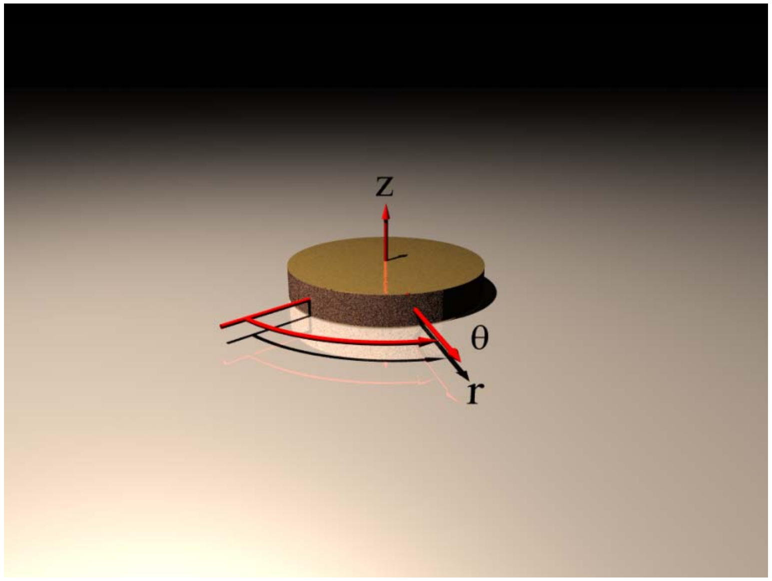
- Formulate axisymmetric analytical coupled actuator-structural guided wave excitation model for surface-bonded piezoelectric actuators on isotropic plates using:
  - 3-D elasticity equations for structure[1]
  - IEEE Standard piezoelectricity equations for actuator[2]
  - "Pin-force" approximation to couple the two models

- Verify formulation using finite element analysis (FEA)

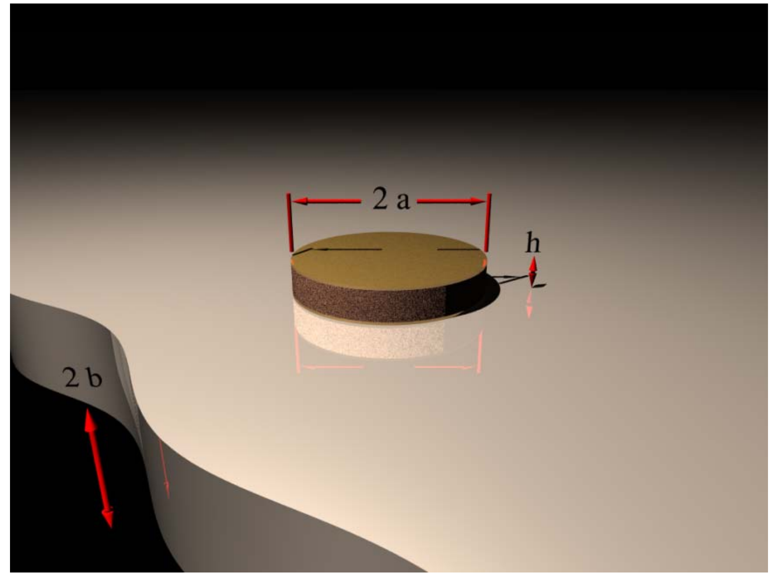
1 Raghavan and Cesnik, “Finite dimensional piezoelectric transducer modeling for guided wave based structural health monitoring,” Smart Materials and Structures, 2005

2 “IEEE Standard on Piezoelectricity”, Institute of Electrical and Electronics Engineers, Inc, ANSI/IEEE Std 176-1987

Problem Formulation



Axes



Geometry

- Piezoelectric actuator bonded to an infinite isotropic plate
- Cylindrical coordinate system used, z-axis along thickness
- Actuator poled along z-axis, electroded on the z surfaces
- Actuator driven by sinusoidal voltage
- Structure and actuator joined at $r = a$

Structure Model

- Start with 3-D elasticity equations simplified for axisymmetry
- Apply Hankel integral transform and assume harmonic excitation at angular frequency ω
- Matching the transformed normal stresses at the plate surfaces ultimately yields, for surface radial displacement:

$$u_r = \tau_{Str}(\omega)\left(\sum_{\xi=\xi^A,\xi^S} aJ_1(\xi a)\frac{N_{A/S}(\xi)}{D'_{A/S}(\xi)}H_1^{(2)}(\xi r)e^{i\omega t}\right)$$

where

- ξ is the wavenumber of the free Lamb-wave at frequency ω
- $N_{A/S}$ are functions of ξ & plate material properties
- $D_{A/S}$ are the antisymmetric/symmetric Lamb-wave equations
- τ_{Str} is the magnitude of shear traction exerted by the actuator
- $J_n(\ )$ is the Bessel function
- $H_n(\ )$ is the Hankel function of order n

Actuator Model

- ANSI/IEEE 176-1987 standard axi-symmetric thin piezoelectric disk solution extended to include radial forcing at the edges
  - assumes no electric field fringing
  - assumes independent of the z direction variations
  - assumes stress free through the thickness and on the edges

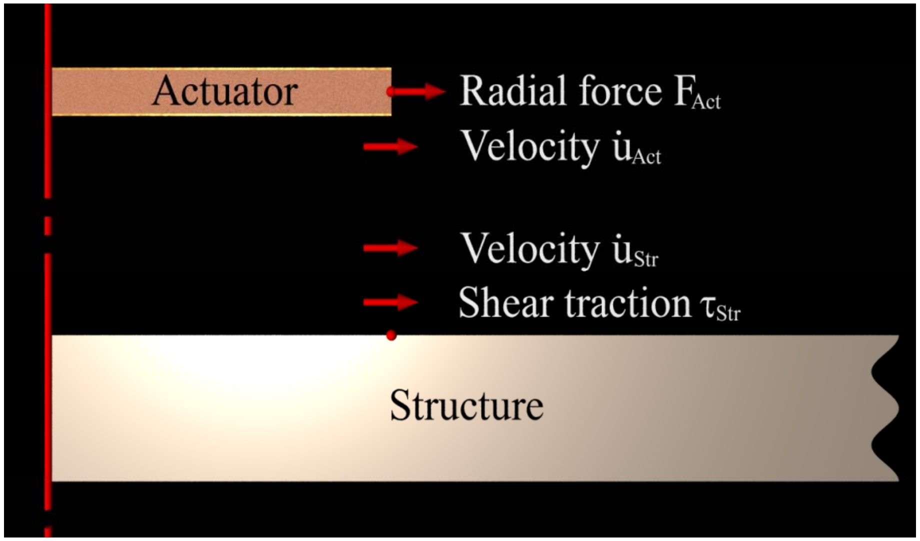
- For a disk with edge radial force F_{act} & edge radial velocity $\dot{u}_{Act}$:

$$\begin{Bmatrix} F_{Act}(\omega) \\ V_{Act}(\omega) \end{Bmatrix} = \begin{bmatrix} Z_{11}^{Act}(\omega) & Z_{12}^{Act}(\omega) \\ Z_{12}^{Act}(\omega) & Z_{22}^{Act}(\omega) \end{bmatrix} \begin{Bmatrix} \dot{u}_{Act}(\omega) \\ i_{Act}(\omega) \end{Bmatrix}$$

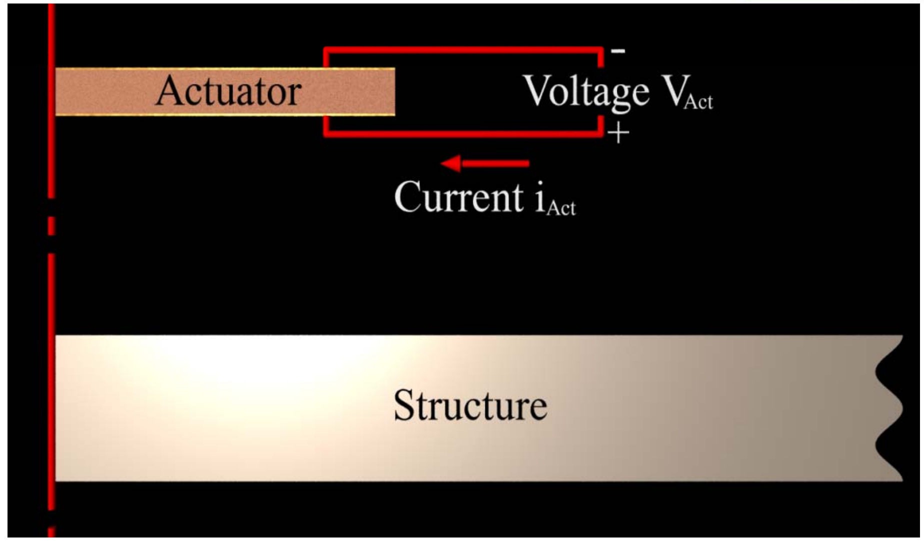
where

- Z_{ij}^{Act} are impedance matrix elements (functions of material & geometry)
- V_{act} is the applied voltage
- i_{Act} is the current drawn by the actuator

Coupling Approximation



Velocity, forces, and tractions



Voltage and current

- From "pin-force" model[1], surface-bonded actuator transfers traction only along its edge on the structure
- Two models assumed to be coupled at the edge $r = a$
- Force balance $\Rightarrow$ $F_{Act}(t) = -2\pi a\,\tau_{Str}(t)$
- Continuity of displacement $\Rightarrow$ $\dot{u}_{Act}(t) = \dot{u}_{Str}(t)$

1. Crawley and de Luis, "Use of piezoelectric actuators as elements of intelligent structures," AIAA Journal, 1987

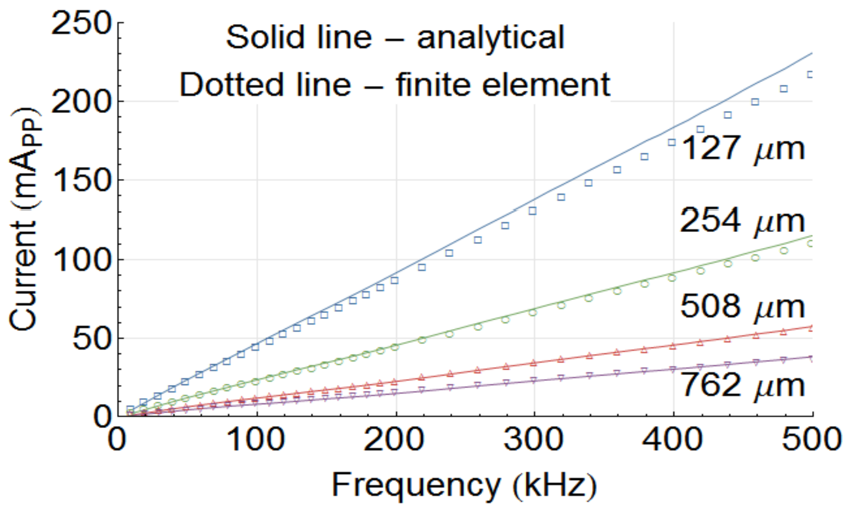
Axisymmetric Finite Element Model

- Finite Element Modeling performed in Ansys^{TM} 11.0
- Response simulated in the time domain
  - large structure chosen to prevent edge reflections (models infinite plate)
  - sampling frequency ten times actuation frequency
  - actuator voltage & charge, displacement recorded at each time step
- Element types
  - actuator: plane 223 (8 node piezoelectric)
  - structure: plane 82 (8 node structural)

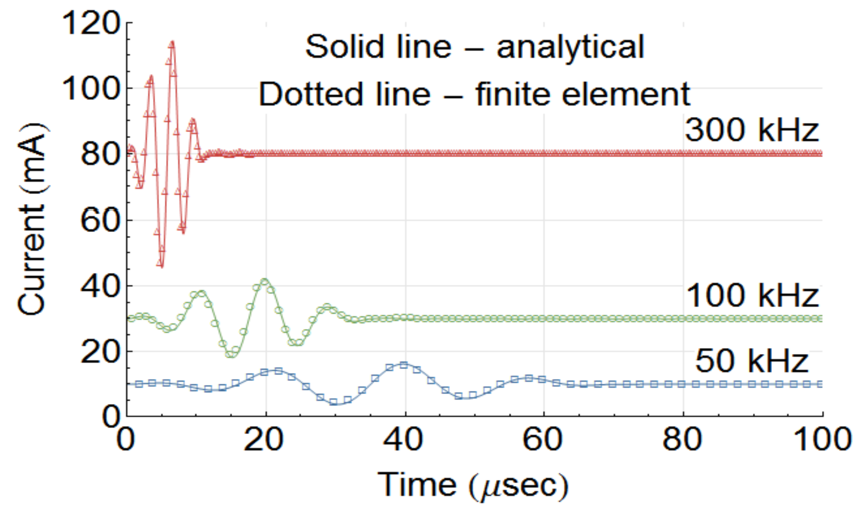
Simulations

- Analytical solution
  - derived in the frequency domain
  - discrete inverse Fourier transform used to convert to time domain
- Finite element solution
  - solved in the time domain
  - peak to peak solutions of time domain data used for comparison
- Actuator
  - PZT-5A
  - 2.54 cm (1”) diameter × 254 μm thick (unless noted)
- Structure
  - aluminum 6061-T6
  - 3.2 mm (0.125”) thick
- Forcing
  - 3.5 cycle 1 V amplitude sinusoid under a Hanning window
  - center frequency varied from 10 to 500 kHz

Actuator Current



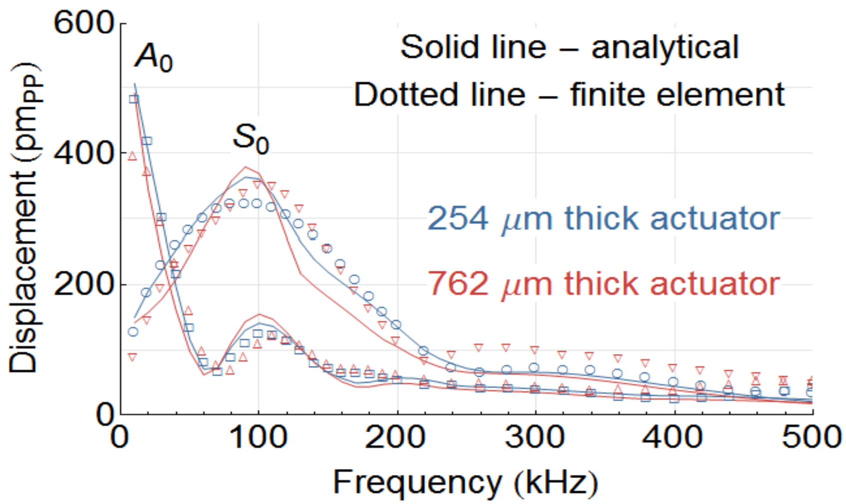
Current versus center frequency for various thickness actuators



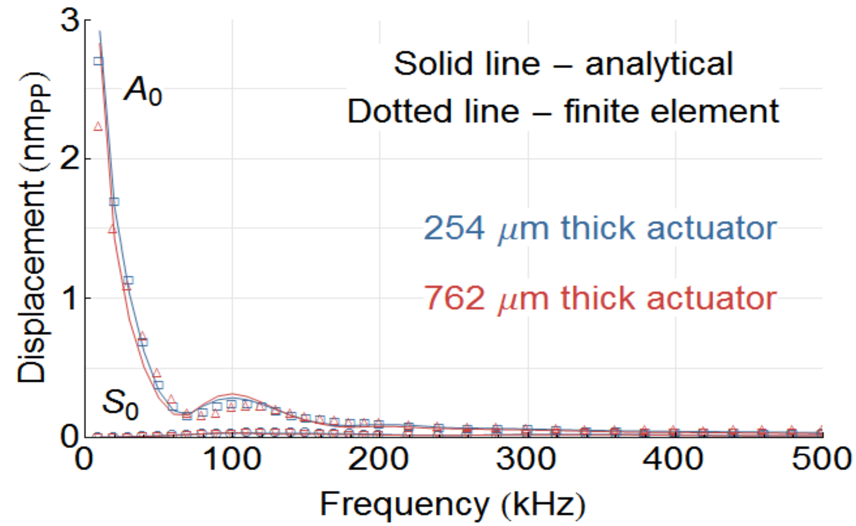
Current versus time

- Frequency domain
  - excellent agreement between models
  - < 6% error seen in the slope
- Time domain
  - excellent agreement between models
  - DC offset added for clarity

Displacement vs Center Frequency



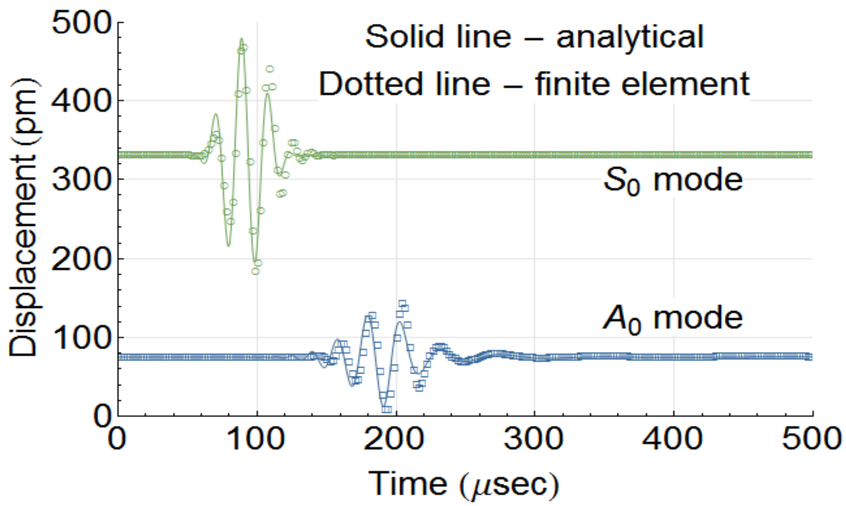
In-plane displacement



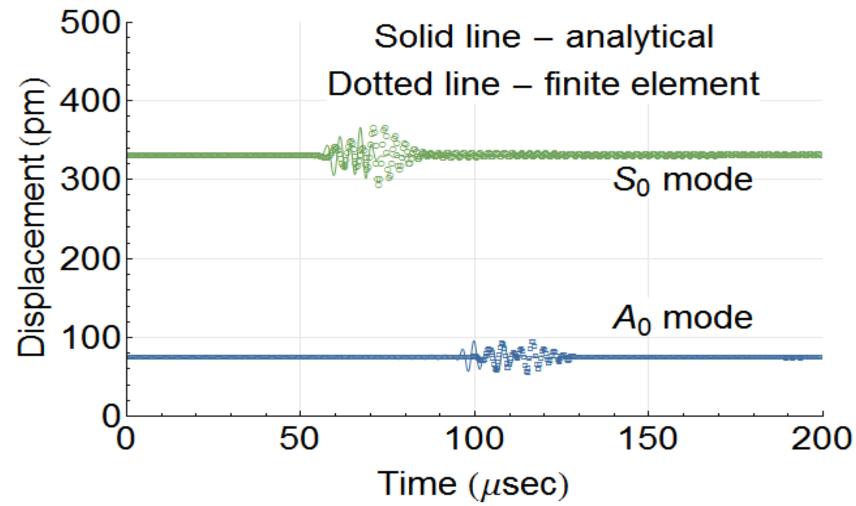
Out-of-plane displacement

- Displacements measured at r = 30 cm
- Good agreement for A_0 mode
- S_0 mode
  - fair agreement between models
  - less agreement near resonance & above 200 kHz
- S_0 & A_0 mode finite element results calculated by taking sum & difference of displacement on either free surface

In-Plane Displacement vs Time



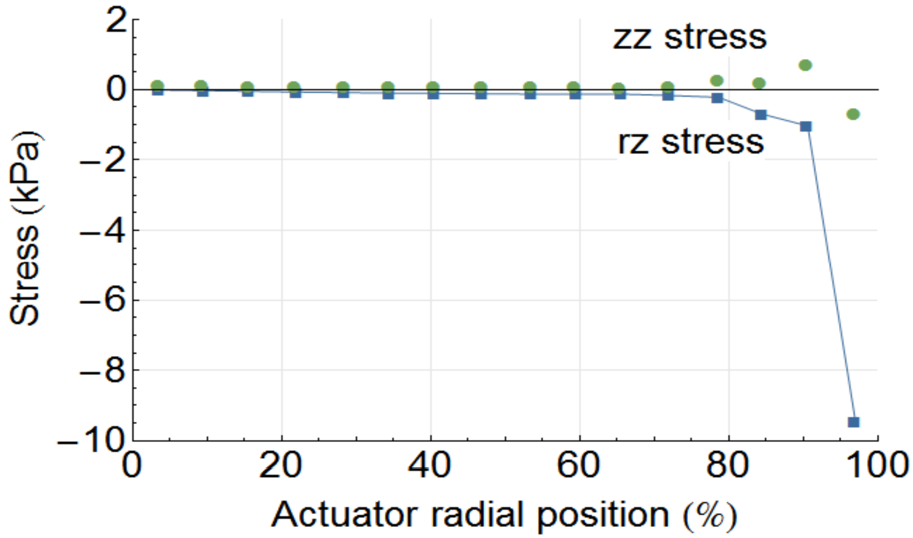
50 kHz actuation



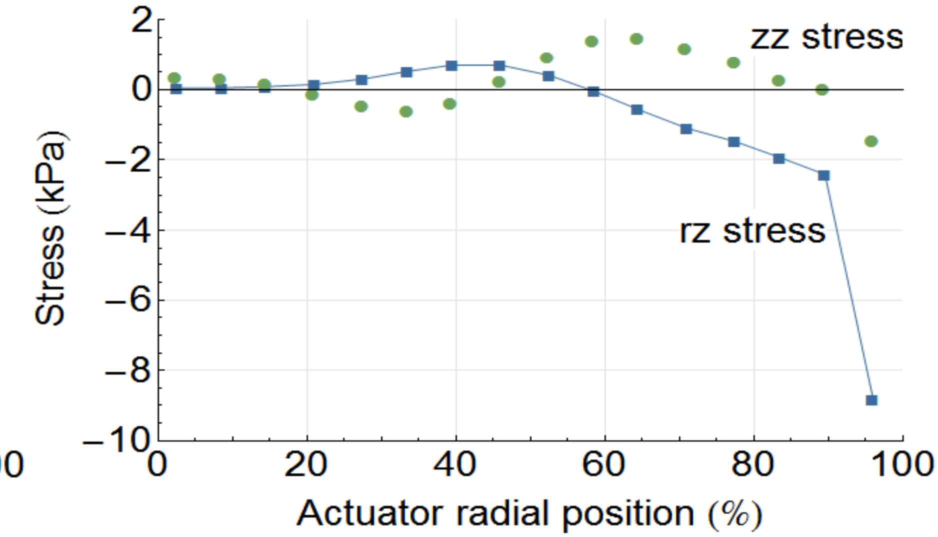
300 kHz actuation

- Displacements measured at r = 30 cm
- DC offset added for clarity
- Good agreement for 50 kHz actuation
- 300 kHz actuation
  - ➢ good magnitude agreement
  - ➢ analytical model data time shifted ~20 μsec with respect to FEM

Discussion of Pin Assumption



50 kHz actuation



300 kHz actuation

- Stresses at actuator / structure interface from FEM plotted at time of peak voltage
- At low frequencies "pin-force" model assumptions hold
  - stress approximately zero except near edge
  - normal stress small compared to shear
- Pin-force model assumptions less valid at higher frequencies
  - edge stress still dominate
  - interior stress smaller but not negligible

Model Utilization

- Determine effects of actuator size & materials on SHM design
- Estimate actuator electrical power draw
- Estimate electrical work done for each mode
- Changes due to bonding to structure in actuator characteristics
  - resonant frequency
  - capacitance
- Estimating sensor response magnitudes & monitoring range
  - must chose desired signal-to-noise ratio
  - need material damping characteristics

Conclusions

- New analytical model developed that captures
  - piezoelectric actuator dynamics
  - all possible guided-wave modes in structure
  - does not require calibration for actuator traction
- Good agreement with FEM given simple assumptions
- Can be used as an effective design tool for SHM system design
- Offers significant advantages in computation time over FEA
  - 2-3 *minutes* for analytical model to process full range of frequencies
  - 2-3 *days* for FEM to process corresponding range

Future Work

- Add *zz* component of stress to coupling
  - would model bending stiffness difference at $r = a$
  - requires a higher fidelity model for actuator
- Include effect of adhesive between actuator & sensor
- Extend model for ring-shaped or rectangular actuator
- Incorporate material damping for sensor density estimation
- Compare to experimental data