Application of Model Assisted Probability of Detection (MAPOD) to a Guided Wave SHM System

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IWSHM-2017

ABSTRACT

This paper updates continued development on a study to generate Probability of detection (PoD) and Receiver Operating Characteristic (ROC) curves for a guided wave (GW) Structural Health Monitoring (SHM) system. PoD curves are used to access a detector's performance as a function of damage size or equivalently the backscattered Energy to Noise Ratio (ENR). ROC curves present statistical representations of the reliability of the method for detecting certain size flaws versus their Probability of False Alarms (PFA). To a large extent, generating meaningful versions of these curves involves multiple repetitions of the same experiments while injecting as much realistic variability as possible. This paper examines the impact of a few key variables such as averaging and distance to flaw and how they can be applied to model assisted PoD.

INTRODUCTION

SHM systems are permanently integrated within a structure to efficiently detect, locate, and characterize damage. This process involves an information extraction procedure where measurements are processed to determine if damage is present. For GW methods, inspection consists of interrogating the structure with an input excitation and then analyzing the resulting scattered wave field using a signal processing method (detector). In this paper, a single GW mode phased array detector is considered.

Before an SHM systems can be fielding in military or commercial applications, its performance as detector needs to be characterized. While no standards presently exist that are specifically written for characterizing SHM performance, best practice suggests that MIL-HDBK-1823 "Non-Destructive Evaluation System Reliability Assessment", can be leveraged due to the close relationship between many NDE and SHM. The key figure of merit from HDBK-1823A is the PoD(a) curve, which is the probability of detection as a function of damage size (a). Resulting from the PoD(a) curve is the $a_{90/95}$

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value, which is the damage size that can be detected with a 90% probability of detection and a 95% confidence bound. An important implication for SHM in establishing a PoD(a) curve is the required number of independent test specimens. For SHM applications, testing of many specimens is typically impractical due to the expense associated with the permanent nature of SHM sensor installation. Model assisted probability of detection (MAPOD) then becomes vital to establishing PoD curves for SHM applications. This includes the incorporation of propagation, scattering, and operational/environmental models with experimental data to properly establish PoD. Initial sources of variability include but are not limited to temperature, strain, humidity, installation location, bond quality, and sensor element degradation.

A previous paper had presented a preliminary PoD curve established using simulated cracks on a metallic plate structure for a guided wave detector. This paper builds on previous work by examining how key variables can be calibrated then applied using MAPOD strategies to determine the effect on PoD. Specifically, the effect of various numbers of averages and distance between the sensor and damage are explored.

ACTIVE ULTRASONIC GUIDED WAVE INSPECTION

Pulse echo detection

A pulse echo strategy detects damage by interrogating the structure with a waveform and detecting the scattered/echo from damage. The pulse echo detection problem is mathematically modeled in discrete time as:

$$H_{0} = \overset{w[n]}{\underset{A_{0} \cos(2\pi f_{0}n + \phi) + w[n]}{m = n_{0}, n_{0} + 1, \dots, n_{0} + M, \dots, N - 1}} \\ H_{1} = \overset{w[n]}{\underset{A_{1} \cos(2\pi f_{0}n + \phi) + w[n]}{m = n_{0}, n_{0} + 1, \dots, n_{0} + M, \dots, N - 1}} \\ H_{\kappa} = \overset{w[n]}{\underset{A_{\kappa} \cos(2\pi f_{0}n + \phi) + w[n]}{m = n_{0}, n_{0} + 1, \dots, n_{0} - 1, n_{0} + M, \dots, N - 1}} \\ n = 0, 1, \dots, n_{0} - 1, n_{0} + M, \dots, N - 1 \\ \vdots \\ n = 0, 1, \dots, n_{0} - 1, n_{0} + M, \dots, N - 1 \\ n = n_{0}, n_{0} + 1, \dots, n_{0} + M - 1 \end{bmatrix}$$
(1)

Here H_{κ} is the K^{th} damage state/size with echo amplitude A_{κ} , frequency f_0 , phase ϕ , arrival time n_0 and noise w[n]. Assuming the noise w[n] is a random variable with known probability distribution function (PDF), each damage state is modeled as a multivariate PDF. Damage detection is a multiple hypothesis testing problem; damage not present (H_0) and damage present (H_{κ}) . Classification is accomplished through multiple hypothesis testing; decide between $(H_0, H_1, H_2, \dots, H_{\kappa-1}, H_{\kappa})$. The design of detectors, or the decision rule that decides the proper hypothesis for a given state, is dependent upon the parameters of the signal and corresponding PDF.

Binary Hypothesis Testing

A binary hypothesis test results from Equation (1) when, $A_k = 0$ for $k \neq 0,1$. This model simulates the problem of deciding if damage is or is not present. Simple binary hypothesis testing, is deciding between two hypotheses where the PDFs' parameters

under each hypothesis are known. The Neyman-Pearson method says to maximize the probability of detection P_D for a given probability of false alarm $P_{FA} = \alpha$, decide H_1 if

$$L(x) = \frac{p(\mathbf{x}; H_1)}{p(\mathbf{x}; H_0)} > \gamma , \qquad (2)$$

where the threshold γ is found from

$$P_{FA} = \int_{\{\mathbf{x}: L(\mathbf{x}) > \gamma\}} p(\mathbf{x}; H_0) d\mathbf{x} = \alpha.$$
Binary Hypothesis Test
(3)



Figure 1: Binary hypothesis testing.

where $\hat{\theta}_1$ is the maximum likelihood estimate (MLE) of θ_1 assuming H_1 is true (maximizes $p(\mathbf{x}; \theta_1, H_1)$), and $\hat{\theta}_0$ is the maximum likelihood estimate of θ_0 assuming H_0 is true (maximizes $p(\mathbf{x}; \theta_0, H_0)$).

Signal Model

An array of *M* transducers located at arbitrary position $\mathbf{p}_m = \begin{bmatrix} p_{x_m} & p_{y_m} \end{bmatrix}^T$, samples the spatiotemporal field for backward or forward scattering from damage with inter-element transducers spacing sufficiently small to allow phase coherent processing and avoid affects from spatial aliasing. The scattering source is assumed located several wavelengths away from the array center, and any scattering received by the array approximated as a plane wave with wavevector, $\mathbf{k}^{\{\alpha\}}$, where the superscript $\alpha = 1, 2...P$ represents the scattered wave mode. A scattered signal received at the m^{th} transducer and sampled discretely in time is mathematically modeled as

$$\tilde{s}_{m}[n] = \sum_{\alpha=1}^{P} A^{\{\alpha\}} \exp\left[\phi^{\{\alpha\}}\right] \Pi^{\{\alpha\}}[n,r] \exp\left[j2\pi f^{\{\alpha\}}n\right] \exp\left[-j2\pi f^{\{\alpha\}}\tau_{m}^{\{\alpha\}}\right]$$
(4)

where $A^{\{\alpha\}}$ is amplitude, $\phi^{\{\alpha\}}$ is absolute phase relative to the scattering source, $f^{\{\alpha\}}$ is temporal frequency, and $\tau^{\{\alpha\}}$ is the relative inter-element phase delay due to the direction of

propagation for given a wavevector. An observation is composed of N samples ranging from n = 0...N - 1. The inter-element phase delay referenced to the array center for mode α , propagating in direction β , is

$$\tau_m^{\{\alpha\}} = -\frac{\mathbf{p}_m^T}{c_p^{\{\alpha\}}} \left[\cos(\beta) \quad \sin(\beta) \right]^T, \tag{5}$$

where $c_p^{\{\alpha\}}$ is the phase velocity. The function $\Pi^{\{\alpha\}}[n,r]$, is a boxcar function that represents the time when wave mode α , scattered from a source a distance *r* away, is present at all array transducers,

$$\Pi^{\{\alpha\}}[n,r] = H\left[n - n^{\{\alpha\}}\right] - H\left[n - \left(n^{\{\alpha\}} + K^{\{\alpha\}}\right)\right],\tag{6}$$

where $n^{\{\alpha\}} = r/c_g^{\{\alpha\}}$ is the arrival sample, $c_g^{\{\alpha\}}$ is group velocity and $K^{\{\alpha\}}$ is the length of the signal. A single mode signal model is given as

$$\tilde{s}_{m}[n] = A^{A_{0}} \exp\left[\phi^{A_{0}}\right] \Pi^{A_{0}}[n,r] \exp\left[j2\pi fn\right] \exp\left[-j2\pi f\tau_{m}^{A_{0}}\right]$$
(7)

Detector Derivation: location, amplitude, phase unknown

It is assumed that amplitude, absolute phase, and scattering location for the signal model are deterministic but unknown and that the excitation frequency, phase velocity and group velocity are known. This represents an active sensing scheme where a known actuation signal interrogates a plate structure of known dispersion relation for damage of unknown size, and location. Determining the presence of damage is equivalent to a binary composite hypothesis test where under the null hypothesis, H_0 , only noise is present and under the alternative hypothesis, H_1 , noise and scattering from damage is present. The observed array data under each hypothesis is a vector random variable and described by a probability density function (PDF). Letting \tilde{x} represent the observed array data vector, the PDF under H_0 is represented as $p(\tilde{x}; H_0)$ and under H_1 as $p(\tilde{x}; \theta_1, H_1)$, where θ_1 is a vector of the unknown parameters. To decide between the two hypothesizes, the unknown parameters are first estimated and then used in a generalized likelihood ratio test (GLRT). The GLRT decides H_1 if

$$L_{G}\left(\tilde{\mathbf{x}}\right) = \frac{p\left(\tilde{\mathbf{x}}; \boldsymbol{\theta}_{1}, H_{1}\right)}{p\left(\tilde{\mathbf{x}}; H_{0}\right)} > \gamma', \qquad (8)$$

where $\hat{\theta}_1$ is the maximum likelihood estimate (MLE) of θ under H_1 , the value of θ_1 that maximizes $p(\tilde{\mathbf{x}}; \theta_1, H_1)$. The detector assuming only a single propagating mode model is given as

$$T(\tilde{\mathbf{x}}) = \max_{r,\beta} \frac{2}{\sigma^2 \left(\left(KM \right)^2 \right)} \left[\left| \mathbf{h}_1^H \tilde{\mathbf{x}} \right|^2 \right].$$
(9)

See (Jarmer, Flynn and Todd 2013) for a derivation and description of the detector.

PROBABILITY OF DETECTION

Theoretical POD Curves

The probability of detection and false alarm for the theoretical detector is given as

$$P_{FA} = Q_{\chi_2^2}(\gamma'), P_D = Q_{\chi_2^{(2)}(\lambda_1)}(\gamma').$$
(10)

 λ_1 is equal to the energy to noise ratio (ENR),

$$\lambda_{1} = \frac{A_{1}e^{-j\phi_{1}}MNA_{1}e^{j\phi_{1}}}{\sigma_{2}^{2}} = \frac{A_{1}^{2}MN}{\sigma_{2}^{2}}$$

 $Q_{\chi_2^2}$ is the right tail probability for a central Chi-Squared PDF with 2 degrees of freedom and similarly $Q_{\chi_2^2(\lambda)}$, is the right tail probability for a noncentral Chi-Squared PDF with 2 degrees of freedom and noncentrality parameter λ_1 . Figure 2 is a plot of probability of detection as a function of ENR for false alarm rates ranging 1e-1 to 1e-6. POD increases with ENR level and with decreasing false alarm rate.

The probability of detection is thus a function of the energy to noise ratio which allows for the incorporation of model assisted POD through the signal amplitude and noise parameters. For example the signal amplitude can be modeled as an exponential decay when propagating through complex structural features such as longerons. Figure 3 shows experimental data and an exponential decay model for an aerospace structure consisting of wave propagation through multiple longerons in both flat and curved fuselage sections.

Incorporating this model, we explicitly write the probability of detection as a function of inspection distance, I_p , as

$$P_{D}(I_{D}) = Q_{\chi_{2}^{2}(\lambda_{1})}(\gamma').$$
(11)

Figure 4 shows the resulting receiver operating characteristic (ROC) curves; as the inspection distance increases the performance of the detector decreases due to the resulting attenuation of the waveform.

In a similar manner, the variance for the uncorrelated Gaussian noise, σ^2 , can be parameterized as a function of repeated averages. In theory, uncorrelated Gaussian noise variance is reduced by a factor of $1/N_{Ave}$ where N_{Ave} is the number of averages. Figure 5 shows the theoretical reduction in noise variance along with experimental data whereby repeated data sets were collected for averages ranging from 1 to 4096. Figure 6 illustrates the effect of averaging on a time series trace. Figure 7 is an illustrative ROC curve where the ENR ratio is modified to show the effects of increasing number of averages as even for a single actuation the POD can be very high. For active guided wave SHM applications, white Gaussian noise is not the primary factor which dictates POD. Instead the main influencing factor is in-frequency band noise caused by imperfect baseline subtraction.







Figure 3: Theoretical and experimental attenuation profiles.



Figure 4: ROC curve as a function of inspection distance.



Figure 5: Theoretical and experimental reduction of noise variance for increasing number of averages.



Figure 6: Reduction of uncorrelated noise variance due to increasing number of averages.



Figure 7: Illustrative ROC curve parameterized by number of signal averages.

CONCLUSIONS

During prior work, experimental PoD and ROC curves were generated by Monte Carlo methods using a large number of experimental test runs for a notch in an aluminum sandwich panel. This paper focused on the use of Model Assisted Probability of Detection (MAPOD) to examine the implications of various variables on the generated PoD and ROC curves. The effects of distance to damage and number of averages collect were empirically calibrated, and the resulting PoD and ROC curves were presented. Future planned work will focus on the effect of non-ideal variables on PoD. This includes the testing of specimens under varying temperature and strain in addition to investigation of other factors of influence such as repeatability of sensor bond line and independence of repeated sampling on an individual test specimen. PoD and ROC curves are essential for assessing the reliability of SHM methods, which must be conducted in order to field these technologies within any commercial or military application.

ACKNOWLEDGMENTS

This research was performed at the Metis Design Corporation in Boston, MA, and sponsored by Navy SBIR contracts: N68335-13-C-0226 & N68335-17-C-0184 and Air Force SBIR contracts: FA8501-13-C-0023 & FA8501-15-C-0025.

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