Application of Model Assisted Probability of Detection (MAPOD) to a Guided Wave SHM System

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structural health monitoring
multi-functional materials
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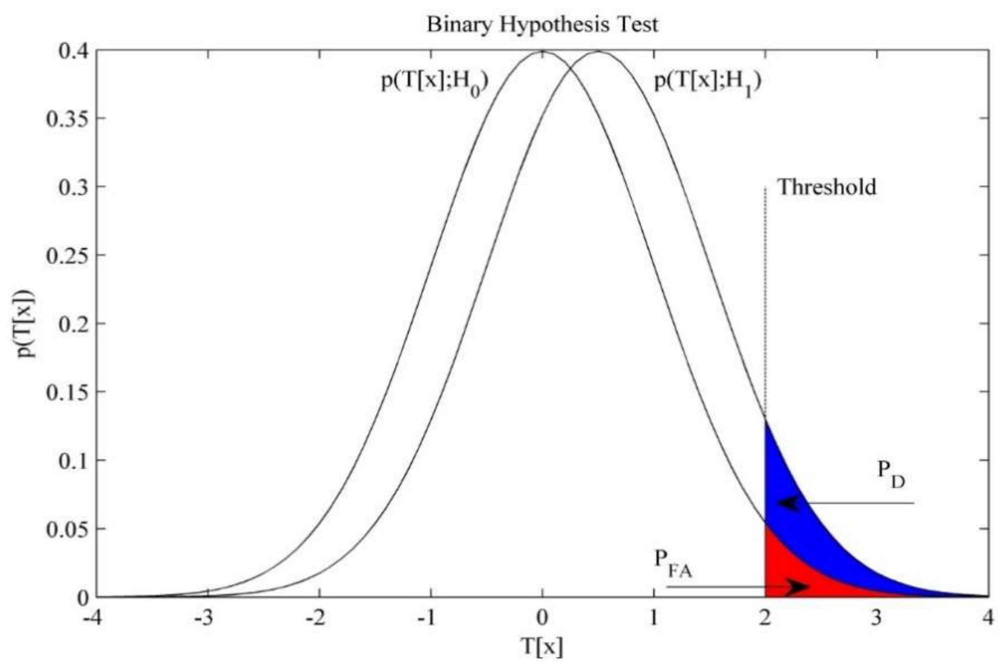
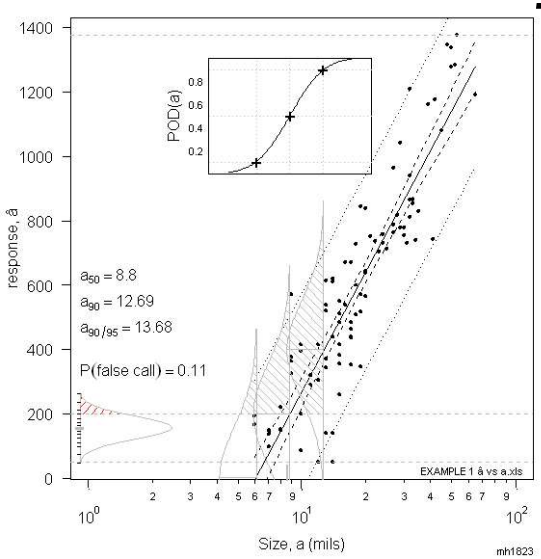
What is Probability of Detection?

- Binary hypothesis test
  - X is data
  - T[x] is signal processing algorithm ("detector")
  - P(T[x]) is probability density function for a given state H

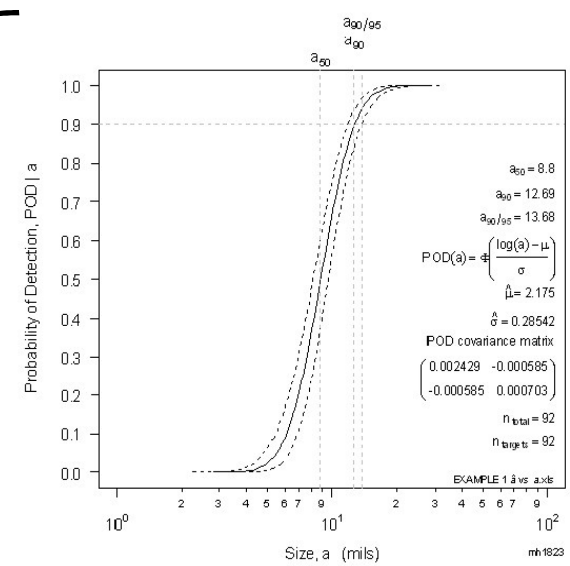


Damage Tolerant Design & Probability of Detection

- Damage tolerant approach common for DoD/commercial design
  - ➢ Requires definition of minimum detectable flaw size
  - ➢ Requires inspection interval set to find minimum flaw with safety factor
- $a_{90/95}$ is flaw size found 90% of the time with 95% confidence
  - ➢ MIL-HDBK 1823A establishes guidelines for NDE reliability assessment
  - ➢ Probability of Detection (PoD) method presented to determine $a_{90/95}$



Experimental Data



PoD Curve

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PoD for Structural Health Monitoring

- **SHM methods differ from NDE methods**
  - SHM sensors are generally integrated into a structure permanently
  - SHM monitors an area, not a point: PoD a function of distance/orientation
  - Sources of variability: bonding, temperature, loading conditions, ect.

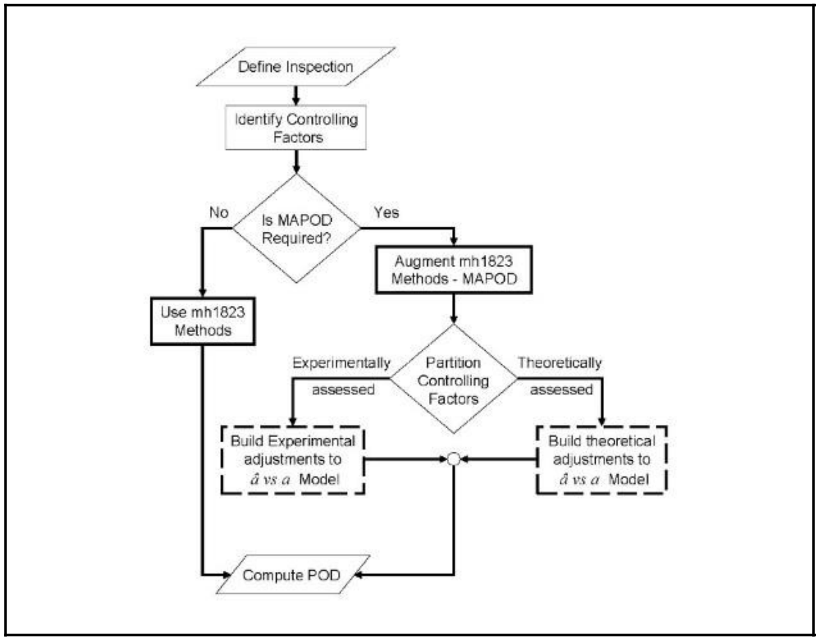
- **Difficult to establish POD for SHM for all variations of parameters**

- **Model Assisted POD (MAPOD) becomes essential**

- *Must use model assisted POD (MAPOD) for SHM systems*
  - *Sources of variability are integrated into experimentally updated models*

MAPOD

- **MAPOD allows use of theoretical models to compute POD**
  - **Traditional 1823a only experimental based**
  - **Theoretical models can include:**
    - —wave attenuation in bulk material due to change in inspection distance
    - —change of material type (AL 6061 to AL 7075) etc.



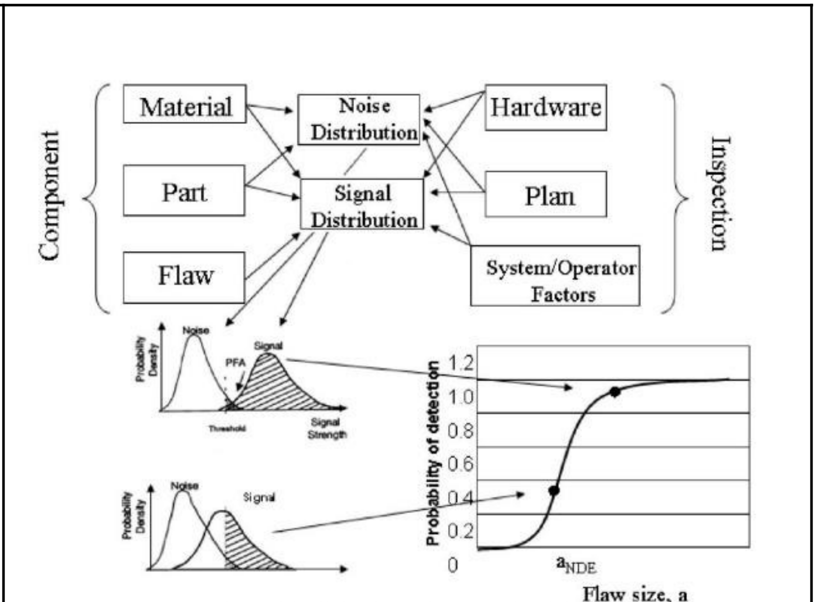
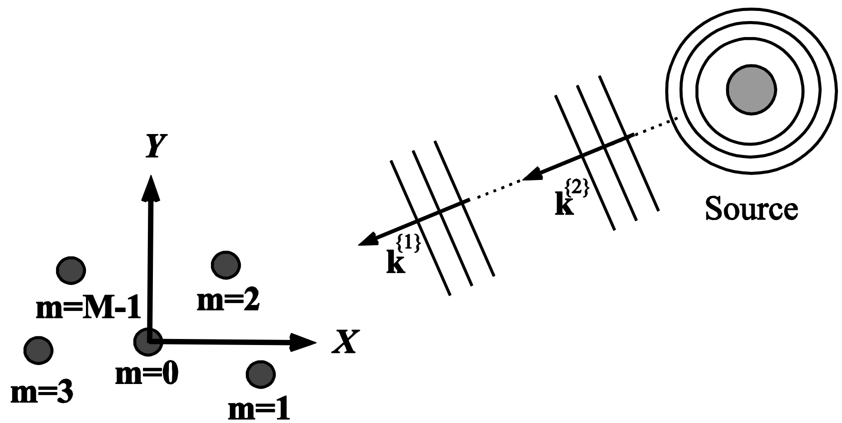


Figure 2. Full Model-Assisted Approach to POD

*Taken from Recent Advances in Model-Assisted POD

Active Sensing Guided Wave Model (1)

- Assume signal model



$$\tilde{s}_m[n] = A^{A_0} \exp\left[\phi^{A_0}\right] \Pi^{A_0}[n,r] \exp[j2\pi fn] \exp\left[-j2\pi f \tau_m^{A_0}\right]$$

- The signal at the senor m is narrow band
  - Phase shift due to inter-element spacing as multiplying factor

Active Sensing Guided Wave Model (2)

- Under state of noise only

$$p\left(\tilde{\mathbf{x}};H_0\right)$$

- Under state of signal plus noise

$$p\left(\tilde{\mathbf{x}};\boldsymbol{\theta}_1,H_1\right)$$

- Generalized Likelihood ratio test: maximize POD for fixed false alarm rate

$$L_G\left(\tilde{\mathbf{x}}\right)=\frac{p\left(\tilde{\mathbf{x}};\boldsymbol{\theta}_1,H_1\right)}{p\left(\tilde{\mathbf{x}};H_0\right)}>\gamma'$$

- Resulting detector is

$$T\left(\tilde{\mathbf{x}}\right)=\frac{2}{\sigma^2\left(\left(KM\right)^2\right)}\left[\left|\mathbf{h}_1^H\tilde{\mathbf{x}}\right|^2+\left|\mathbf{h}_2^H\tilde{\mathbf{x}}\right|^2\right]$$

where

$$\left|\mathbf{h}_1^H\tilde{\mathbf{x}}\right|^2=\left|\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\tilde{x}_m\left[n\right]\Pi^{A_0}\left[n,r\right]\exp\left[-j2\pi f\left(n-\tau_m^{A_0}\right)\right]\right|^2$$

Active Sensing Guided Wave Model (3)

- Resulting closed form statistics of detector are Chi-squared distributed

  - Probability of false alarm detection $P_{FA} = Q_{\chi_2^2}(\gamma')$

  - Probability of detection $P_D = Q_{\chi_2'^2(\lambda_1)}(\gamma')$

  - Import parameter is the non-centrality parameter

$$\lambda_1 = \frac{A^2 MN}{\sigma^2/2}$$

  - This is the energy to noise ratio (ENR)

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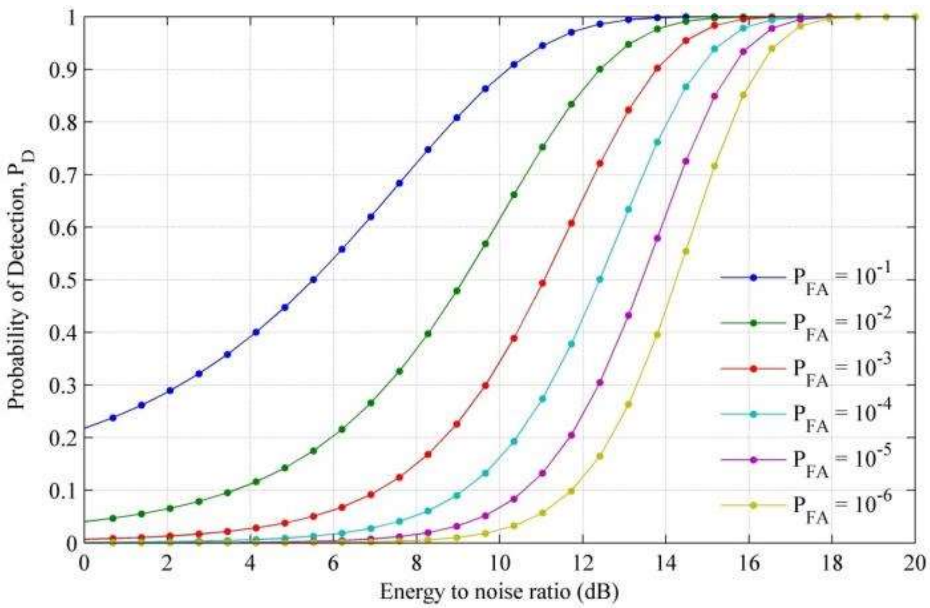
Active Sensing Guided Wave Model (4)

- POD curve is results of varying the SNR ratio

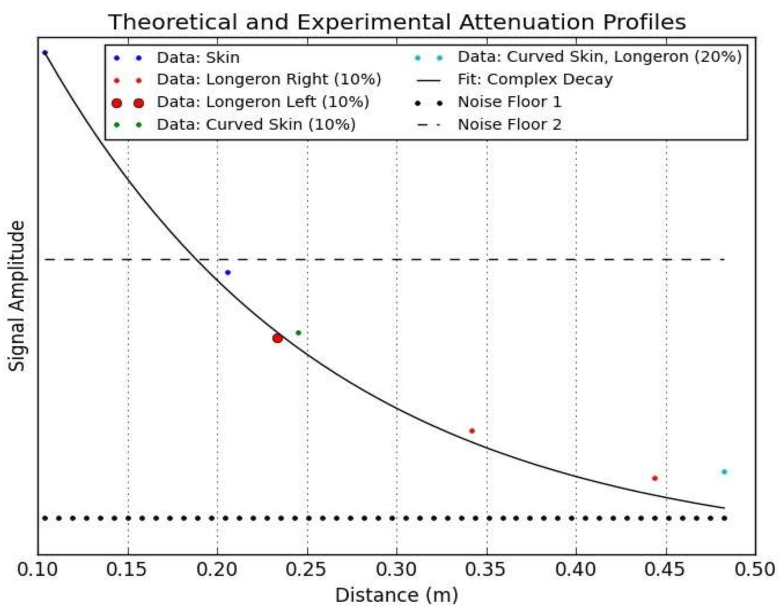


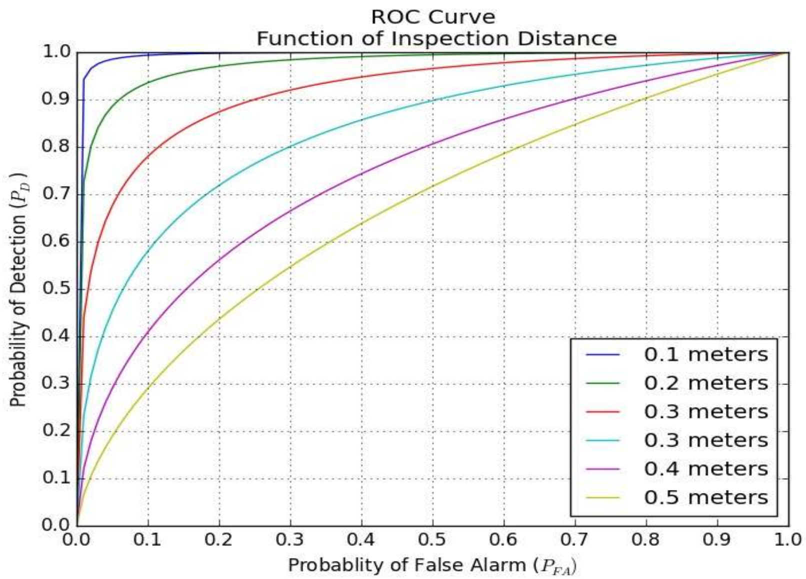
$$\lambda_1 = \frac{A^2 MN}{\sigma^2 / 2}$$

- Assuming "far" scattering field
  - Parameterize A as $A(\mathrm{x})$
  - Scattering amplitude is now a function of the material attenuation profile
  - Allows for calculation of POD as a function of distance to damage

Attenuation Model

- **Incorporate theoretical attenuation to transform POD as a function of distance**
  - ➢ **Model updating of attenuation profile with experimental data**





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Application parameters

- **Under what conditions can this be applied**
  - Only when underlying sources of variability remain unchanged
  - Change in like material type
    - Aluminum to steel,
  - Decrease in uncorrelated noise environment
    - Upgraded A/D

- **Under what conditions can this <u>NOT</u> be applied**
  - Aluminum to carbon fiber
  - Changes in material thickness
    - Unless scattering of new wave number is shown to scale accordingly
    - Also there cannot be a change to PZT actuator/sensor resonant characteristics
  - Signal model is not valid
    - Correlated noise
    - Orthogonality of guided wave modes

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Summary

- Differentiation between SHM & NDI for reliability assessment
  - SHM permanently installed at a point to cover an area, cost driver
  - Different variables effect PoD (temperature), no operator
- MAPOD becomes essential in order to implement POD for SHM
  - Reduce physical test matrix size with models to minimize cost
- Basic guided wave signal model results in detector with closed form statistics
  - POD is driven by ENR or SNR which correlates to signal amplitude
- Implementation of MAPOD techniques is application specific

Future Work

- Building analytical tools under NAVAIR SBIR funding in order to facilitate MAPOD for guided wave SHM in metallic structures

- Conducting validation experiments for guided wave SHM
  - Metallic structures for crack growth
  - Composite structures for delamination

- Main focus initially on demonstration of validity of distance to damage MAPOD approach for metallic applications
  - Next will look at influence of geometry
  - Then will investigate limits of applicability with thickness changes

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